$\mathbb{R}^n$ Calc Test

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1.

a.

The distance to the xz-plane is the y coord of the point. The point with the smallest magnitude y-coord is $(6, -1, 2)$, B.

b.

The distance from the origin is

$$\begin{aligned}\|(x, y, z)\| &= \sqrt{x^2 + y^2 + z^2}\\ \|A\| &= \sqrt{32}\\ \|B\| &= \sqrt{41}\\ \|C\| &= \sqrt{26}\end{aligned}$$

The smallest one is C.

c.

The distance between two points is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

$$\begin{aligned}\overline{AB} &= \sqrt{34}\\ \overline{AC} &= \sqrt{77}\\ \overline{CB} &= \sqrt{83}\end{aligned}$$

Side CB is the longest with length $\sqrt{83}$

2.

a.

$$z^2 + y^2 = 9$$

b.

i. $z = \pm 3$

ii. Can't be done. The cylinder cuts through all planes $||$ to the yz plane.

iii. $y = \pm 3$

c.

We want to find the shortest distance between the point $(5, 7, 9)$ and (x, y, z) which lies on the cylinder. Obviously, $x = 5$, since we can set x as we please. The distance between the points will be:

$$\sqrt{(7 - y)^2 + (9 - z)^2}$$

From the equation, $z=\sqrt{9-y^2}$. Thus, the distance is

$$\sqrt{(7-y)^2+(9-\sqrt{9-y^2})^2}$$

Using the calculator, we find that the minimum of this function is 8.40...

3.

$$D(p_1,p_2)=D_0+\frac{\Delta D}{\Delta p_1}(p_{1,0}-p_1)+\frac{\Delta D}{\Delta p_2}(p_{2,0}-p_2)$$
$$D_{\text{push}}=70,000+\frac{20}{-4,000}(300-p_1)+\frac{20}{600}(1,200-p_2)$$
$$D_{\text{ride}}=19,000+\frac{-100}{10,000}(300-p_1)+\frac{20}{5,000}(1,200-p_2)$$

4.

a.

The limit does not exist. At the point $(0,0)$, contour lines of different values converge. Each contour line represents a different approach towards the point $(0,0)$. The different approaches give different z values. Thus, no limit exists.

b.

The limit does exist. There is only one value given by any approach to $(0,0)$. Different contour lines are not converging at that point; only one is. Thus, the limit exists.